

Fig. 11
The equation of the curve shown in Fig. 11 is $y=x^{3}-6 x+2$.
(i) Find $\frac{\mathrm{d} y}{\mathrm{~d} x}$.
(ii) Find, in exact form, the range of values of $x$ for which $x^{3}-6 x+2$ is a decreasing function.
(iii) Find the equation of the tangent to the curve at the point $(-1,7)$.

Find also the coordinates of the point where this tangent crosses the curve again.

2 Find $\frac{\mathrm{d} y}{\mathrm{~d} x}$ when $y=x^{6}+\sqrt{x}$.

3 (i) Find the equation of the tangent to the curve $y=x^{4}$ at the point where $x=2$. Give your answer in the form $y=m x+c$.
(ii) Calculate the gradient of the chord joining the points on the curve $y=x^{4}$ where $x=2$ and $x=2.1$.
(iii) (A) Expand $(2+h)^{4}$.
(B) Simplify $\frac{(2+h)^{4}-2^{4}}{h}$.
(C) Show how your result in part (iii) (B) can be used to find the gradient of $y=x^{4}$ at the point where $x=2$.

4 (i) Calculate the gradient of the chord joining the points on the curve $y=x^{2}-7$ for which $x=3$ and $x=3.1$.
(ii) Given that $\mathrm{f}(x)=x^{2}-7$, find and simplify $\frac{\mathrm{f}(3+h)-\mathrm{f}(3)}{h}$.
(iii) Use your result in part (ii) to find the gradient of $y=x^{2}-7$ at the point where $x=3$, showing your reasoning.
(iv) Find the equation of the tangent to the curve $y=x^{2}-7$ at the point where $x=3$.
(v) This tangent crosses the $x$-axis at the point P . The curve crosses the positive $x$-axis at the point Q . Find the distance PQ , giving your answer correct to 3 decimal places.


Fig. 12

Fig. 12 shows part of the curve $y=x^{4}$ and the line $y=8 x$, which intersect at the origin and the point $P$.
(A) Find the coordinates of P , and show that the area of triangle OPQ is 16 square units.
(B) Find the area of the region bounded by the line and the curve.
(ii) You are given that $\mathrm{f}(x)=x^{4}$.
(A) Complete this identity for $\mathrm{f}(x+h)$.

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\begin{equation*}
\mathrm{f}(x+h)=(x+h)^{4}=x^{4}+4 x^{3} h+\ldots \tag{2}
\end{equation*}
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(B) Simplify $\frac{\mathrm{f}(x+h)-\mathrm{f}(x)}{h}$.
(C) Find $\lim _{h \rightarrow 0} \frac{\mathrm{f}(x+h)-\mathrm{f}(x)}{h}$.
(D) State what this limit represents.

